ent decreases the critical distance but increases the critical Reynolds number. It is also seen that increasing surface tension increases both Re_c and x_c and has a stabilizing effect.

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APPENDIX

The authors extend Ünsal and Thomas' studies [5] and correctly consider the surface tension variation caused by the interfacial temperature fluctuation through wave motion. The calculated Re_c is derived as

$$Re_c = \left[\frac{(\frac{4}{3}\xi N_\sigma)^{1/2}}{\frac{8}{15}-\frac{1}{2}\,WeCa}\right]^{6/11}. \tag{A1}$$

When the surface tension is uniform (i.e. $Ca = 0$), the calculated Re_c shown in the following equation is the same as the result obtained by Ünsal and Thomas.

$$Re_c = \left[\frac{75}{16}\xi N_\sigma\right]^{3/11}. \tag{A2}$$

The critical distance is derived as

$$x_c = \frac{1}{4}\left[\left(\frac{3\rho}{64\sigma_0}\right)^4 \nu^{-2} g^5 (1-\gamma)^5 \left(\frac{8}{15}-\frac{1}{2}\,WeCa\right)^8 \xi^7\right]^{-1/11}. \tag{A3}$$

Similarly, for $Ca = 0$, eqn (A3) becomes

$$x_c = \frac{1}{4}\left[(75\sigma_0)^4 \left(\frac{c_p}{k\zeta}\right)^7 \frac{\rho^3 \nu^9}{g^5(1-\gamma)^5}\right)\Bigg]^{1/11}. \tag{A4}$$

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Correction functions for a wide range of measured substrate temperature histories

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INTRODUCTION

As highlighted by a number of recent studies [1–2], the design of high temperature components, ceramics in particular, is a difficult process because of the often conflicting thermal and mechanical criteria that define their service. These "conflicts" will no-doubt increase as machines and materials are pushed to their strength and temperature limits in the search for improved efficiency and cost effectiveness. Because of the increasingly severe conditions that components are expected to endure, proactive design methodologies using predictive thermal and stress models are necessary to expose and explore all threats to component safety during typical service conditions. Yet, any realistic modeling of these potentially

NOMENCLATURE

L	substrate thickness
$R(t)$	measured thermocouple response
r	thermocouple wire radius
t	Fourier number like dimensionless time, $\alpha_{Sb}t^*/r^2$
t^*	dimensional time
a_0-a_3	integral-order polynomial coefficients
b_1-b_3	half-order polynomial coefficients
C_1	dimensionless constant, $\beta/(8/\pi^2+\beta)$
C_2	dimensionless constant, $4/(8/\pi+\beta\pi)$
C_3	dimensionless constant, C_2-1.

Greek symbols

$\Delta T(t)$	substrate temperature forcing function
$\Phi(t)$	unit response of an intrinsic thermocouple
$\Psi(t)$	function defined by equation (15)
α	thermal diffusivity ratio, α_{Tc}/α_{Sb}
β	dimensionless constant, $\kappa/\sqrt{\alpha}$
$\delta_{a0}-\delta_{a3}$	integral-order correction functions
$\delta_{b1}-\delta_{b3}$	half-order correction functions
κ	thermal conductivity ratio, k_{Tc}/k_{Sb}.

Subscripts

Sb	substrate
Tc	thermocouple.

complex service conditions will usually require precise temperature measurements taken from actual components and/or scaled prototypes.

Although many temperature measurement techniques exist, surface mounted thermocouples represent one of the most practical means of monitoring surface temperatures over a wide range of values and complex geometries. Nevertheless, the use of thermocouples are not without difficulties because of the deleterious influences of the mass of the thermocouple, contact resistance and the problems of debonds from the surface [3], as well as stray heat transfer to the surrounding environment. To offset the potentially significant errors that may result from any combination of these factors a number of useful, albeit empirically based, thermocouple corrective measures have been developed over the years [4–6]. Unfortunately, these methods required extensive calibration and testing (with their own inherent errors) to determine the correction factors. Hence, the accuracy and practicality of these methods for industrial applications can become limited.

Fortunately, more recent analytical solutions using Laplace transforms [7–8] have offered significant improvements to the resolution of errors without extensive calibration and reliance on empirical constants. Nonetheless, their utility in correcting the measurement of the complex temperature histories seen in industrial service is limited in part, by the assumptions pertaining to how the substrate temperature is changing with times. Hence, while all of the corrective measures discussed above have been helpful in analyzing and correcting temperature data, the usefulness of the solutions is ultimately limited by their ability to accurately describe increasingly complex heat transfer events.

The purpose of this paper is, therefore, to derive correction functions for intrinsic thermocouples with a response modeled by a versatile polynomial based on lower-order integral- and half-order powers of time that should encompass a wide range of substrate temperature histories. Furthermore, to facilitate the adaptation of the solutions to an arbitrary measured temperature history, the correction functions are derived individually for each half- and integral-order polynomial component.

ANALYTICAL CONSIDERATIONS

The problem at hand is to derive useful analytical solutions for a thermally thick ($L/r > 10$) substrate temperature history based on the response measured by an intrinsic thermocouple (i.e. no junction height) as shown by Fig. 1. Under the assumption of constant thermal properties for the thermally thick substrate and thermocouple, the principal of superposition may be used to relate the thermocouple's response $R(t)$, to an arbitrary substrate temperature loading rate $\Delta T(t)$, through the use of Duhamel's integral:

$$R(t) = \frac{\partial}{\partial t}\int_0^t \Delta T(\tau)\cdot\Phi(t-\tau)\,\mathrm{d}\tau \qquad (1)$$

where $\Phi(t)$ is the kernel and represents the area-averaged response of a thermocouple to a unit step temperature change and the variable t represents a Fourier number like dimensionless time parameter. For an intrinsic thermocouple without contact resistance and no appreciable stray heat transfer to the surrounding environment, the late time ($t > 0.1$) unit response for the thermocouple is defined as [7]:

$$\Phi(t) = 1-C_1\exp(C_2^2t)[1-\mathrm{erf}(C_2\sqrt{t})] \qquad (2)$$

where

$$C_1 = \beta/(8/\pi^2+\beta) \qquad (3)$$

and

$$C_2 = 4/(8/\pi+\beta\pi). \qquad (4)$$

An estimation of the actual substrate temperature history may then be obtained by first approximating the measured response $R(t)$, as the sum of any combination of integral- and half-order polynomial terms (a_j and b_j) in non-dimensional time t, such that:

$$R(t) = a_0+\sum_{j=1}^{m}[a_jt^j+b_jt^{j-1/2}]. \qquad (5)$$

Restricting the range of m to 3 and the corresponding highest powers of the integral and half-order terms to 3 and 5/2, respectively, to minimize the likelihood of any unstable curve oscillations around individual data points, a general solution may then be realized by substituting equations (2) and (5) into equation (1), taking the Laplace transform and rearranging terms:

$$\Delta T(s) = \frac{R(s)}{s\cdot\Phi(s)}. \qquad (6)$$

A substrate forcing function can then be directly calculated if equation (6) can be inverted either analytically or numerically. Fortunately, the direct inversion of equation (6) with all of the individual components of equation (5) is possible and yields the following general expression for a substrate

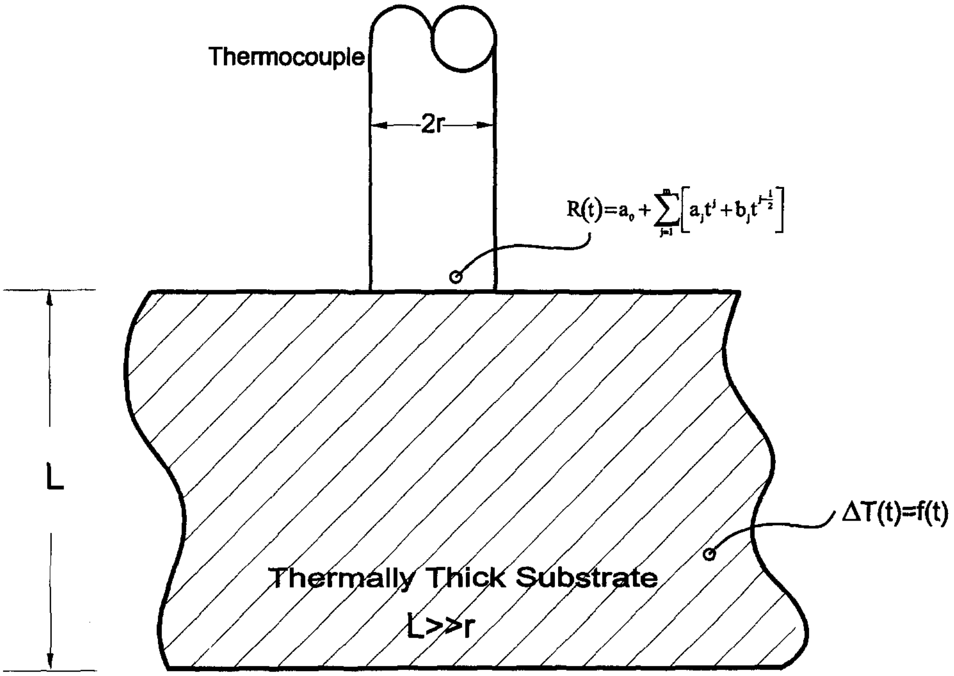

Fig. 1. Idealized intrinsic thermocouple and thermally thick substrate geometry without contact resistance or stray heat transfer to the surrounding environment.

surface temperature as defined by the polynomial coefficients.

$$\Delta T(t) = a_0\delta_{a0}(t) + \sum_{j=1}^{3}[a_j\delta_{aj}(t) + b_j\delta_{bj}(t)]. \tag{7}$$

For equation (7), the polynomial based correction functions (δ_{aj} and δ_{bj}) are defined as:

$$\delta_{a0}(t) = 1 + \frac{1}{C_2}\left[\frac{C_1}{\sqrt{\pi t}} - C_3(1+C_3)\Psi(t)\right] \tag{8}$$

$$\delta_{a1}(t) = t + \frac{C_1}{C_2^2}\left[C_3 + \frac{C_3^2}{C_2\sqrt{\pi t}} + \frac{2C_2\sqrt{t}}{\sqrt{\pi}} - \frac{C_3^3}{C_2}\Psi(t)\right] \tag{9}$$

$$\delta_{a2}(t) = t^2 + \frac{2C_1}{C_2^2}\left[\frac{C_3^3}{C_2^2} + C_3 t + \frac{C_3^4}{C_2^3\sqrt{\pi t}} + \frac{2C_3^2\sqrt{t}}{C_2\sqrt{\pi}} + \frac{4C_2 t^{3/2}}{3\sqrt{\pi}} - \frac{C_3^5}{C_2^3}\Psi(t)\right] \tag{10}$$

$$\delta_{a3}(t) = t^3 + \frac{6C_1}{C_2^2}\left[\frac{C_3^5}{C_2^4} + \frac{C_3^3 t}{C_2^2} + \frac{C_3 t^2}{2} + \frac{C_3^6}{C_2^5\sqrt{\pi t}} + \frac{2C_3^4\sqrt{t}}{C_2^3\sqrt{\pi}} + \frac{4C_3^3 t^{3/2}}{3C_2\sqrt{\pi}} + \frac{16C_2 t^{5/2}}{30\sqrt{\pi}} - \frac{C_3^7}{C_2^5}\Psi(t)\right] \tag{11}$$

$$\delta_{b1}(t) = \sqrt{t} + \frac{C_1\sqrt{\pi}}{2C_2^2}\left[C_2 + \frac{C_3}{\sqrt{\pi t}} - C_3^2\Psi(t)\right] \tag{12}$$

$$\delta_{b2}(t) = t^{3/2} + \frac{3C_1\sqrt{\pi}}{4C_2^2}\left[\frac{C_3^2}{C_2} + C_2 t + \frac{C_3^3}{C_2^2\sqrt{\pi t}} + \frac{2C_3\sqrt{t}}{\sqrt{\pi}} - \frac{C_3^4}{C_2^2}\Psi(t)\right] \tag{13}$$

$$\delta_{b3}(t) = t^{5/2} + \frac{15C_1\sqrt{\pi}}{8C_2^2}\left[\frac{C_3^4}{C_2^3} + \frac{C_3^2 t}{C_2} + \frac{t^2}{2C_2} + \frac{C_3^5}{C_2^4\sqrt{\pi t}} + \frac{4C_3^3\sqrt{t}}{C_2^2\sqrt{\pi}} + \frac{4C_3 t^{3/2}}{\sqrt{\pi}} - \frac{C_3^6}{C_2^4}\Psi(t)\right] \tag{14}$$

where the recurring term $\Psi(t)$ seen in equations (8)–(14) can be simplified to contain the exponentially-scaled error function:

$$\Psi(t) = \exp\left(\frac{C_2^2 t}{C_3^2}\right)\left[1 - \operatorname{erf}\left(\frac{C_2\sqrt{t}}{|C_3|}\right)\right] + \frac{1}{C_3\sqrt{\pi t}} \tag{15}$$

provided

$$C_3 = C_1 - 1. \tag{16}$$

DISCUSSION

The derived solutions along with a relatively simple procedure allow the reconstruction of complex substrate temperature histories from surface temperature data measured with intrinsic thermocouples. This procedure requires the use of least-squares techniques to fit a polynomial containing integral- and half-order terms to empirical temperature data. Once the resulting polynomial has been examined for "goodness-of-fit," the coefficients can be used with equations (8)–(15) to calculate the thermocouple correction functions. Equation (7) can then be used to directly estimate the substrate forcing function $\Delta T(t)$.

The ratio of a typical measured response to the calculated substrate forcing function during the thermal shock of a silicon carbide tube [2] is shown in Fig. 2. As shown by the figure, the approximated response of the 0.25 mm diameter, chromel–alumel (type-K) intrinsic thermocouple as modeled by a cubic polynomial is initially slow relative to the thermally shocked substrate. However, the derived response of the thermocouple does asymptotically approach the sub-

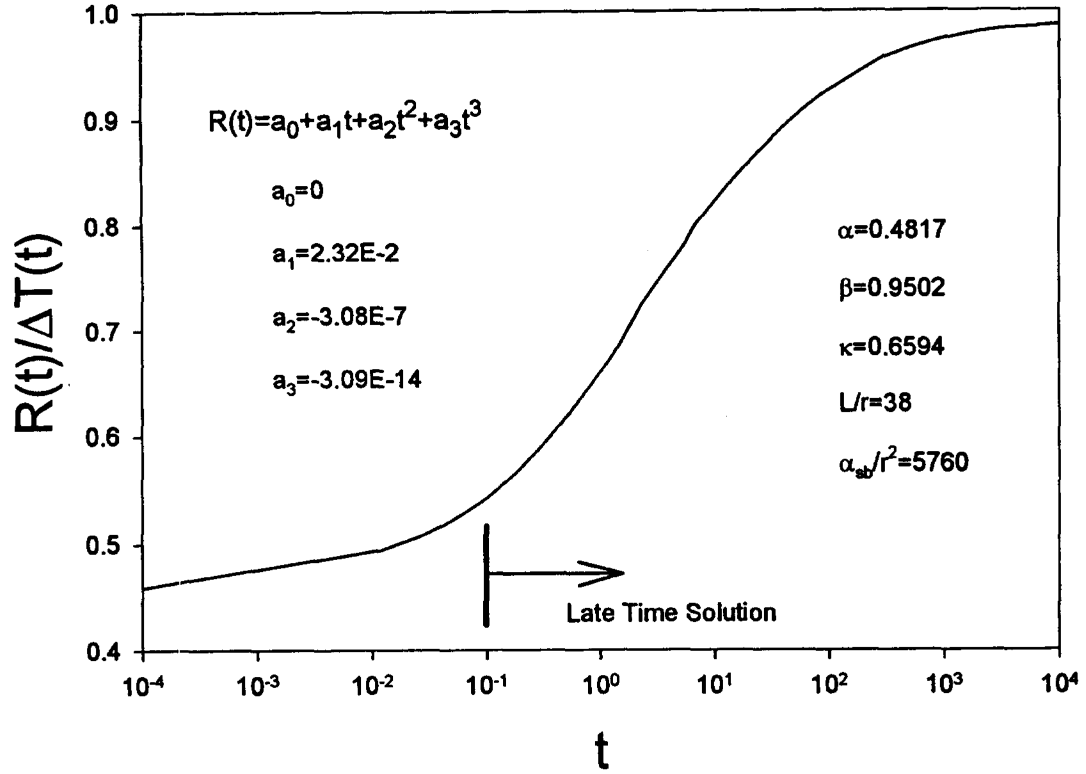

Fig. 2. Typical transient response of a 0.25 mm diameter, chromel–alumel (type-k) thermocouple mounted to a thermally shocked silicon carbide substrate.

strate for increasing values of nondimensional time as steady-state is approached. Interestingly, the response of the thermocouple relative to the calculated substrate forcing function appears to follow the trends predicted by earlier studies [7] for the entire time interval, even though the early solutions ($t < 0.1$) were not used in the derivation.

Although the inversion process used to derive equations (8)–(15) was exact, the ensuing algebraic expression must be considered approximate at best because of the reliance on interpolating polynomials. In most cases, the validity of the temperature data corrections will be predicated on the accuracy of the polynomial fit, as well as the soundness of the assumptions pertaining to the absence of both contact resistance and lateral heat transfer to the surrounding environment. However, the significance of the approximation may be offset by the possibility that a carefully applied polynomial will also tend to smooth out experimental errors that could come from a large number of independent sources.

While this study was restricted to maximum polynomial powers of 3 and 5/2 for integral- and half-order terms, respectively, higher-order terms may also be readily evaluated in closed form. However, the unstable curve oscillations around individual data points associated with higher-order polynomials may begin to adversely influence the corrections. This may also be true if unconstrained, piece-wise polynomials such as splines are used to interpolate the measured temperature histories. In these instances, it may be advisable to use smoothed or tensioned splines. Although not explored in this study, alternate substrate histories defined by exponentials or periodic functions may also be directly or numerically inverted.

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